Uncertainty Principle Respects Locality

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The notion of nonlocality implicitly implies there might be some kind of spooky action at a distance in nature, however, the validity of quantum mechanics has been well tested up to now. In this work it is argued that the notion of nonlocality is physically improper, the basic principle of locality in nature is well respected by quantum mechanics, namely, the uncertainty principle. We show that the quantum bound on the Clauser, Horne, Shimony, and Holt (CHSH) inequality can be recovered from the uncertainty relation in a multipartite setting, and the same bound exists classically which indicates that nonlocality does not capture the essence of quantum and then distinguish quantum mechanics and classical mechanics properly. We further argue that the super-quantum correlation demonstrated by the nonlocal box is not physically comparable with the quantum one, as the result, the physical foundation for the existence of nonlocality is falsified. The origin of the quantum structure of nature still remains to be explained, some post-quantum theory which is more complete in some sense than quantum mechanics is possible and might not necessarily be a hidden variable theory.

I. INTRODUCTION

The origin of the notion of nonlocality dates back to the Einstein, Podolsky, and Rosen (EPR) argument [1], which claimed that quantum mechanics is not complete, and as well a kind of hidden variable theory is expected to explain the incompleteness of quantum mechanics. Significant breakthrough was made by Bell [2] who discovered inequalities which are named after him, and later on Clauser, Horne, Shimony, and Holt (CHSH) [3], these inequalities could be employed to rule out local hidden variable theories, and thus demonstrate nonlocality. Despite the wide acceptance of quantum nonlocality, the arguments both on philosophical and physical issues persist [4–7], see Ref. [6] and references therein. Generally, the notion of locality means a physical system in spacetime exists and contains independent properties of itself with respect to the rest of universe, despite there is no standard definition of this notion in literatures. Then, nonlocality would indicate that a system loses its self-dependence and there could be some kind of "spooky action at a distance", which makes the essence of quantum mechanics even more mysterious. To understand this, it is later on realized quantum system does not violate the CHSH inequality maximally, and a super-quantum correlation follows from the so-called nonlocal box, or Popescu-Rohrlich (PR) machine [8]. Along with the development of nonlocality, the notion of entanglement [9] also significantly attracts the interest of physicists. It is well recognized that nonlocality is different from entanglement, however, "what is nonlocality?" is still not a well answered question.

We attempt to explain nonlocality by the uncertainty principle, which lies at the innermost place of quantum mechanics. Standard uncertainty relation is expressed with expectation values of observable [10–12], some new kinds of relations are derived concerning entropy and measurement settings, namely, the entropic uncertainty relation [13–15]. Entropic inequality for nonlocality which originally involves expectation values of observable is also developed [16]. Recently there are attempts to relate nonlocality to entropic uncertainty principle and steering [17], here we establish the connection between uncertainty principle and nonlocality directly. Our conclusion in this work is that nonlocality is not a physically valid concept, we demonstrate our claim via three steps. First, we generalize the uncertainty relation to multipartite system, from which, the quantum Tsirelson bound [18] is recovered, which means the quantum correlation manifested by CHSH inequality follows directly from uncertainty principle. The connection between the quantum bound and uncertainty principle particularly the Cauchy-Schwarz inequality [19] is noticed before [20, 21], yet the physical implication is not discussed in details, namely, the physical validity of nonlocality is not questioned. Secondly, regarding the classical bound, which is 2 for the CHSH inequality, we find that there exists one different way to carry out the classical correspondence of CHSH operator, and the new bound equals $2\sqrt{2}$ which is the same as the quantum bound. The new classical model can be viewed as a classical simulation of the quantum case, which indicates that nonlocality can also exist classically, as the result, the notion of nonlocality becomes trivial. Furthermore, we verify that for the nonlocal box it is not possible to assign consistent local probabilities to the local parts of the box, and the correlation in the nonlocal box is not the same type and thus not comparable with the quantum and even the classical cases. As the result, we are lead to the conclusion that nonlocality is not a physically proper notion, and locality, which is one of the most basic notions in physics, is respected by the uncertainty principle very well.

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This work contains three parts. In section II, we review the uncertainty principle and generalize its formula to multipartite systems. We show that the quantum bound on CHSH inequality follows directly from the uncertainty principle. In section III, we analyze the classical bounds and super-quantum bound in nonlocal box on CHSH inequality. In section IV, we discuss issues on complementarity, the completeness of quantum mechanics, the hidden variable theory, and then conclude.

II. UNCERTAINTY PRINCIPLE

The uncertainty principle is originally established by Heisenberg [10], and then generalized by Robertson and Schrödinger [11, 12]. Note that this principle is a *logical* relation, which does not necessarily relate to the practical situation, such as noise and disturbance in practical measurement [22, 23]. The Cauchy-Schwarz inequality [19, 24], which relies directly on the geometry of the Hilbert space, is employed to derive the uncertainty relation. In this section, we study the uncertainty principle for bipartite system, and the multipartite generalizations follow directly from the fact that there always exists one bipartite partition generally. Then we derive the corresponding quantum bound for CHSH inequality from the uncertainty relation.

For a system $\mathcal{S}$ composed with two subsystems $\mathcal{S}_A$ and $\mathcal{S}_B$, the Hilbert space is bipartite $\mathcal{H} = \mathcal{H}_A^n \otimes \mathcal{H}_B^m$, n and m are the dimensions. Suppose observable A_α and A_β act on $\mathcal{S}_A$, observable B_α and B_β act on $\mathcal{S}_B$, and define collective observable $\tilde{A}_\alpha = A_\alpha \otimes \mathbf{I}$, $\tilde{B}_\alpha = \mathbf{I} \otimes B_\alpha$, etc, identities $\mathbf{I}$ take the corresponding dimensions. The commutation relations are

$$\begin{aligned} [A_\alpha, A_\beta] &\neq 0, \\ [B_\alpha, B_\beta] &\neq 0, \\ [\tilde{A}_i, \tilde{B}_j] &= 0, \end{aligned} \tag{1}$$

where $i, j = \alpha, \beta$. Define two new observable on subsystem $\mathcal{S}_B$ as $P_\alpha = B_\alpha + B_\beta$, $P_\beta = B_\alpha - B_\beta$, then

$$\begin{aligned} [P_\alpha, P_\beta] &\neq 0, \\ [\tilde{A}_i, \tilde{P}_j] &= 0. \end{aligned} \tag{2}$$

To derive the uncertainty relation, there are two quantities we need to evaluate, namely, variance and covariance. The variance of observable, say, P and $\tilde{P}$ are

$$\begin{aligned} \mathrm{Var}(P) &= \langle (\Delta P)^2 \rangle = \langle P^2 \rangle - \langle P \rangle^2, \\ \mathrm{Var}(\tilde{P}) &= \langle (\Delta \tilde{P})^2 \rangle = \langle \tilde{P}^2 \rangle - \langle \tilde{P} \rangle^2, \end{aligned} \tag{3}$$

note the quantum expectations $\langle \cdot \rangle$ are carried out on corresponding states. The covariance, say, between P_α and P_β are

$$\begin{aligned} \mathrm{Cov}(P_\alpha, P_\beta) &= \langle P_\alpha P_\beta \rangle - \langle P_\alpha \rangle \langle P_\beta \rangle, \\ \mathrm{Cov}(P_\beta, P_\alpha) &= \langle P_\beta P_\alpha \rangle - \langle P_\alpha \rangle \langle P_\beta \rangle. \end{aligned} \tag{4}$$

The Cauchy-Schwarz inequality precisely provides the relation between variance and covariance functions, which takes the form

$$|\langle f|g \rangle|^2 \leq \langle f|f \rangle \langle g|g \rangle, \tag{5}$$

for two arbitrary state vectors $|f\rangle$ and $|g\rangle$ in Hilbert space. Let us re-deduce the so-called Schrödinger uncertainty relation [12] for a single system in a more compact way. For subsystem $\mathcal{S}_A$ and observable A_α and A_β, let $\Delta A_\alpha = A_\alpha - \langle A_\alpha \rangle$, and $\Delta A_\beta = A_\beta - \langle A_\beta \rangle$. The general state of subsystem $\mathcal{S}_A$ is in the density matrix form $\rho_A = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, $|\psi_i\rangle \in \mathcal{H}_A^n$. It is much convenient to employ the method of "wave function of ensemble state" [25], namely, define the wave function for $\mathcal{S}_A$ as $|\Psi_A\rangle$, and $\rho_A = |\Psi_A\rangle\langle\Psi_A|$, $|\Psi_A\rangle = \sum_i \gamma_i |\psi_i\rangle$, $|\gamma_i|^2 = p_i$. Then assign the state $|f\rangle = \Delta A_\alpha |\Psi_A\rangle$, and $|g\rangle = \Delta A_\beta |\Psi_A\rangle$, we have

$$\begin{aligned} \langle f|f \rangle &= \mathrm{tr}((\Delta A_\alpha)^2 \rho_A), \\ \langle g|g \rangle &= \mathrm{tr}((\Delta A_\beta)^2 \rho_A), \\ \langle f|g \rangle &= \mathrm{tr}(\Delta A_\alpha \Delta A_\beta \rho_A), \end{aligned} \tag{6}$$

note $\langle f|f \rangle = \langle \Psi_A | (\Delta A_\alpha)^2 | \Psi_A \rangle$. The above equations reduce to $\langle f|f \rangle = \mathrm{Var}(A_\alpha)$, $\langle g|g \rangle = \mathrm{Var}(A_\beta)$, $|\langle f|g \rangle|^2 = \left(\frac{1}{2}\langle \{A_\alpha, A_\beta\} \rangle - \langle A_\alpha \rangle \langle A_\beta \rangle\right)^2 + \left(\frac{1}{2i}\langle [A_\alpha, A_\beta] \rangle\right)^2 = \mathrm{Cov}(A_\alpha, A_\beta)\mathrm{Cov}(A_\beta, A_\alpha)$. Then, from Cauchy-Schwarz inequality we get

$$\mathrm{Cov}(A_\alpha, A_\beta)\mathrm{Cov}(A_\beta, A_\alpha) \leq \mathrm{Var}(A_\alpha)\mathrm{Var}(A_\beta), \tag{7}$$

which can also be expressed as the Schrödinger uncertainty relation

$$\Delta A_\alpha \Delta A_\beta \geq \sqrt{\left(\tfrac{1}{2}\langle \{A_\alpha, A_\beta\} \rangle - \langle A_\alpha \rangle \langle A_\beta \rangle\right)^2 + \left(\tfrac{1}{2i}\langle [A_\alpha, A_\beta] \rangle\right)^2}, \tag{8}$$

and the Heisenberg-Robertson uncertainty relation takes the form

$$\Delta A_\alpha \Delta A_\beta \geq \left| \frac{1}{2i} \langle [A_\alpha, A_\beta] \rangle \right|, \tag{9}$$

note we have used $\Delta A_i = \sqrt{(\Delta A_i)^2}$ to represent the square root of variance of A_i $(i = \alpha, \beta)$ for convenience. At the same time, we can also deduce a kind of uncertainty relation from the triangle inequality. There are two forms of triangle inequalities, the first one is $||f + g|| \leq ||f|| + ||g||$, with the norm $|| \cdot || = \sqrt{\langle \cdot | \cdot \rangle}$. The second one is $|||f|| - ||g||| \leq ||f - g||$, which is equivalent to the first inequality. From triangle inequality we get

$$\frac{1}{4}\left(\mathrm{Cov}(A_\alpha, A_\beta) + \mathrm{Cov}(A_\beta, A_\alpha)\right)^2 \leq \mathrm{Var}(A_\alpha)\mathrm{Var}(A_\beta), \tag{10}$$

from which the inequality (7) based on Cauchy-Schwarz inequality can be derived. This is consistent with the fact that Cauchy-Schwarz inequality is more tight than the triangle inequality.

For subsystem $\mathcal{S}_B$, we can also get inequalities from Cauchy-Schwarz inequality as

$$\begin{aligned}\mathrm{Cov}(P_\alpha, P_\beta)\mathrm{Cov}(P_\beta, P_\alpha) &\le \mathrm{Var}(P_\alpha)\mathrm{Var}(P_\beta),\\ \mathrm{Cov}(B_\alpha, B_\beta)\mathrm{Cov}(B_\beta, B_\alpha) &\le \mathrm{Var}(B_\alpha)\mathrm{Var}(B_\beta),\end{aligned} \tag{11}$$

and from triangle inequality

$$\begin{aligned}&\frac{1}{4}\left((\mathrm{Cov}(P_\alpha, P_\beta) + \mathrm{Cov}(P_\beta, P_\alpha)\right)^2 \le \mathrm{Var}(P_\alpha)\mathrm{Var}(P_\beta),\\ &\frac{1}{4}\left((\mathrm{Cov}(B_\alpha, B_\beta) + \mathrm{Cov}(B_\beta, B_\alpha)\right)^2 \le \mathrm{Var}(B_\alpha)\mathrm{Var}(B_\beta).\end{aligned} \tag{12}$$

Next, following the same method we derive the uncertainty relation for the whole bipartite system $\mathcal{S}$. Suppose the density matrix for the system is $\rho = |\Psi\rangle\langle\Psi|$, and $|\Psi\rangle$ is its wave function. Then define the state $|f_i\rangle = \Delta\tilde{A}_i|\Psi\rangle$, and $|g_j\rangle = \Delta\tilde{P}_j|\Psi\rangle$ for $i, j = \alpha, \beta$, we find

$$\mathrm{Cov}(\tilde{A}_i, \tilde{P}_j)^2 \le \mathrm{Var}(\tilde{A}_i)\mathrm{Var}(\tilde{P}_j), \tag{13a}$$

$$\mathrm{Cov}(\tilde{A}_i, \tilde{B}_j)^2 \le \mathrm{Var}(\tilde{A}_i)\mathrm{Var}(\tilde{B}_j), \tag{13b}$$

in which case, the triangle inequality is equivalent to Cauchy-Schwarz inequality since observable $\tilde{A}_i$ commute with $\tilde{B}_j$ and $\tilde{P}_j$.

The quantum (Tsirelson) bound on CHSH inequality can be derived as follows. From Inequality (13a), we have

$$\langle A_\alpha P_\alpha\rangle + \langle A_\beta P_\beta\rangle \le \sqrt{\mathrm{Var}(\tilde{A}_\alpha)\mathrm{Var}(\tilde{P}_\alpha)} + \sqrt{\mathrm{Var}(\tilde{A}_\beta)\mathrm{Var}(\tilde{P}_\beta)} + \langle\tilde{A}_\alpha\rangle\langle\tilde{P}_\alpha\rangle + \langle\tilde{A}_\beta\rangle\langle\tilde{P}_\beta\rangle, \tag{14a}$$

and

$$\langle A_\alpha P_\alpha\rangle + \langle A_\beta P_\beta\rangle \ge -\sqrt{\mathrm{Var}(\tilde{A}_\alpha)\mathrm{Var}(\tilde{P}_\alpha)} - \sqrt{\mathrm{Var}(\tilde{A}_\beta)\mathrm{Var}(\tilde{P}_\beta)} + \langle\tilde{A}_\alpha\rangle\langle\tilde{P}_\alpha\rangle + \langle\tilde{A}_\beta\rangle\langle\tilde{P}_\beta\rangle, \tag{14b}$$

where we denote $\langle\tilde{A}_\alpha\tilde{P}_\alpha\rangle = \langle A_\alpha \otimes P_\alpha\rangle \equiv \langle A_\alpha P_\alpha\rangle$, etc. Introduce the CHSH operator $\mathfrak{B} = A_\alpha P_\alpha + A_\beta P_\beta$ [3], then the above two inequalities provides the upper bound (14a) and lower bound (14b) for the expectation value of $\mathfrak{B}$ for one certain quantum state. The equality "=" holds for some particular state and observable. One feature of inequality (14) is that it applies to all bipartite quantum states, including entangled, separable, and other types, thus it is not supposed to be employed to witness entanglement directly; on the contrary, it specifies the correlation between observable of the two subsystems under a special state, following from the uncertainty wherein.

In the standard CHSH setting, e.g. one polarization entangled photon pair, we have $\langle A_i\rangle = 0$, $\langle B_i\rangle = 0$, $\langle P_i\rangle = 0$, $A_i^2 = \mathbf{I}$, $B_i^2 = \mathbf{I}$, the spectrum of $\langle A_i\rangle$, $\langle B_i\rangle$ is dichotomic and lives in $[-1, +1]$. As the result, inequality (14) becomes

$$\begin{aligned}|\langle\mathfrak{B}\rangle| &\le \sqrt{\langle\tilde{A}_\alpha^2\rangle\langle\tilde{P}_\alpha^2\rangle} + \sqrt{\langle\tilde{A}_\beta^2\rangle\langle\tilde{P}_\beta^2\rangle}\\ &= \sqrt{2+\lambda} + \sqrt{2-\lambda} \le 2\sqrt{2},\end{aligned} \tag{15}$$

where $\langle\tilde{A}_\alpha^2\rangle = \langle\tilde{A}_\beta^2\rangle = 1$, $\lambda \equiv \langle\{B_\alpha, B_\beta\}\rangle$. The inequality above is the Tsirelson bound [18, 26]. We can verify that the bound is saturated as $2\sqrt{2}$ for the singlet state by operators $\tilde{A}_\alpha = Z\otimes\mathbf{I}$, $\tilde{A}_\beta = X\otimes\mathbf{I}$, $\tilde{B}_\alpha = -\frac{\sqrt{2}}{2}\mathbf{I}\otimes(Z+X)$, $\tilde{B}_\beta = \frac{\sqrt{2}}{2}\mathbf{I}\otimes(Z-X)$, where X, Y, Z are Pauli operators.

In the above we managed to derive the quantum bound on CHSH inequality from uncertainty relation, and thus build the primary connection between uncertainty principle and nonlocality.

III. UNCERTAINTY PRINCIPLE AND NONLOCALITY

A. Classical bound on CHSH inequality

To derive the classical bound on the CHSH inequality, the first step is to map the CHSH operator into one classical quantity. The standard approach is to take the observable as classical dichotomic random variable [3], namely, A_α and A_β as the values for random variable $\mathcal{A}$, also, B_α and B_β for $\mathcal{B}$, and also P_α and P_β for $\mathcal{P}$, respectively. The average $\langle\langle\mathcal{A}\rangle\rangle = 0$, $\langle\langle\mathcal{B}\rangle\rangle = 0$, with $\langle\langle\cdot\rangle\rangle$ as classical average. The absolute value $|A_i| \le 1$, $|B_i| \le 1$ ($i = \alpha, \beta$). It is direct to check that the variance terms in inequality (14) vanishes. In this case, inequality (14) becomes

$$\begin{aligned}|A_\alpha P_\alpha| + |A_\beta P_\beta| &= |A_\alpha||P_\alpha| + |A_\beta||P_\beta|\\ &\le |B_\alpha + B_\beta| + |B_\alpha - B_\beta| \le 2,\end{aligned} \tag{16}$$

which is exactly the classical bound. Historically, it is the classical bound that makes the notion of nonlocality significant and odd. However, there is one crucial difference between the quantum inequality (15) and classical inequality (16), i.e., LHS (left-hand-side) of the quantum bound involves the quantum expectation value of quantum operator, and LHS of the classical bound is the absolute value (instead of classical average) of the entries of classical random variables, this difference leads to the following case.

To derive the classical bound, there is no reason to set observable, say, A_α and A_β correspond to the same

random variable. On the contrary, the reasonable correspondence is to take the four observable as four classical dichotomic random variable, the value also lives in $[-1,+1]$, namely, $\mathcal{A}_\alpha$ for A_α, $\mathcal{A}_\beta$ for A_β, $\mathcal{B}_\alpha$ for B_α, $\mathcal{B}_\beta$ for B_β, also we can introduce $\mathcal{P}_\alpha$ for P_α, and $\mathcal{P}_\beta$ for P_β. The average values of $\mathcal{A}_i$, $\mathcal{B}_i$, and $\mathcal{P}_i$ $(i=\alpha,\beta)$ are all zero. For instance, the average value of $\mathcal{A}_\alpha$ is

$$\langle\langle \mathcal{A}_\alpha\rangle\rangle = \mu_1 a_1^\alpha + \mu_2 a_2^\alpha = 0,$$

with probability $\mu_1+\mu_2=1$, and values $a_1^\alpha, a_2^\alpha \in [-1,+1]$. Note that the sum $a_1^\alpha + a_2^\alpha$ does not necessarily equal to zero. Thus, inequality (14) becomes

$$|\langle\langle \mathcal{A}_\alpha\mathcal{P}_\alpha\rangle\rangle + \langle\langle \mathcal{A}_\beta\mathcal{P}_\beta\rangle\rangle| \leq \sqrt{\langle\langle \mathcal{A}_\alpha^2\rangle\rangle\langle\langle \mathcal{P}_\alpha^2\rangle\rangle} + \sqrt{\langle\langle \mathcal{A}_\beta^2\rangle\rangle\langle\langle \mathcal{P}_\beta^2\rangle\rangle}, \tag{17}$$

with

$$\begin{aligned}\langle\langle \mathcal{P}_\alpha^2\rangle\rangle &= \langle\langle \mathcal{B}_\alpha^2\rangle\rangle + \langle\langle \mathcal{B}_\beta^2\rangle\rangle + 2\langle\langle \mathcal{B}_\alpha\mathcal{B}_\beta\rangle\rangle,\\ \langle\langle \mathcal{P}_\beta^2\rangle\rangle &= \langle\langle \mathcal{B}_\alpha^2\rangle\rangle + \langle\langle \mathcal{B}_\beta^2\rangle\rangle - 2\langle\langle \mathcal{B}_\alpha\mathcal{B}_\beta\rangle\rangle.\end{aligned} \tag{18}$$

It is obvious to see $|\langle\langle \mathcal{B}_\alpha\mathcal{B}_\beta\rangle\rangle| \leq 1$, then with $\langle\langle \mathcal{A}_i^2\rangle\rangle \leq 1$, $\langle\langle \mathcal{B}_i^2\rangle\rangle \leq 1$, the inequality reduces to

$$\begin{aligned}|\langle\langle \mathcal{A}_\alpha\mathcal{P}_\alpha\rangle\rangle + \langle\langle \mathcal{A}_\beta\mathcal{P}_\beta\rangle\rangle| &\leq \sqrt{\langle\langle \mathcal{P}_\alpha^2\rangle\rangle} + \sqrt{\langle\langle \mathcal{P}_\beta^2\rangle\rangle}\\ &\leq \sqrt{2+2\langle\langle \mathcal{B}_\alpha\mathcal{B}_\beta\rangle\rangle} + \sqrt{2-2\langle\langle \mathcal{B}_\alpha\mathcal{B}_\beta\rangle\rangle}\\ &\leq 2\sqrt{2},\end{aligned} \tag{19}$$

which is the same as the quantum bound. The result indicates that this new model can be viewed as a random classical simulation for the quantum case. Strictly speaking, the traditional classical bound on CHSH inequality does not correspond to the quantum case properly, as the result, to define the notion of nonlocality based on the violation of traditional classical bound does not have clear physical motivation and implication. If there were nonlocality, there also exists nonlocality classically, as our study shows above. That is to say, the notion of nonlocality is trivial and misleading.

Recently, along the research line of application of quantum theory to cognitive science [27, 28], a macroscopic model of concept combinations is studied, where the CHSH inequality violation is observed, and then it is claimed there exists entanglement and nonlocality in classical process [29]. In their study, subsystem $\mathcal{S}_A$ is taken as "Animal", subsystem $\mathcal{S}_B$ is taken as "Acts", and the sentence "The Animal Acts" is taken as the whole system. The observable and measuring settings of A (B) are two sets of animals (acts) each again containing two animals (acts). We do not intend to analyze their model in details here, instead, in the spirit of our study above, namely, nonlocality is trivial, we consider the concept combinations model as a classical simulation of the quantum case of CHSH, and therefore, the claim that there is macroscopic entanglement and nonlocality is not correct. Although the quantum correlation can be classically simulated in various ways, the correlations are still in different types. When there are more measuring settings on each quantum subsystems, more random variables corresponding to classical systems are required for simulation, yet there still only is one single bipartite quantum system. That is to say, the bound on the inequality does not actually reveal the essence of quantum, instead, for instance, as well known the most basic property of quantum system is coherence, which is said to be absent for classical system.

Although Bell-type inequalities cannot be used to define nonlocality properly and to detect entanglement directly, it is still of importance. For instance, it is recently realized that the Bell-type inequalities relate to the dimension of the system, and thus can be employed to witness its dimension [30].

B. Nonlocal box

We have shown that the quantum bound on CHSH inequality is a manifestation of uncertainty principle in multipartite system, and the standard physical meaning of nonlocality is misleading. In the following we will further argue that nonlocality is not a physically valid notion.

The CHSH inequality is maximally violated by the nonlocal box, or PR machine, with the bound as 4, without a violation of no-signalling and causality principle [8]. It is not possible to arrive at this bound from the uncertainty relation, and actually, we find that the original model which manifests the maximal violation is inconsistent. As usual, subsystem $\mathcal{S}_A$ performs measurements A and A', subsystem $\mathcal{S}_B$ performs measurements B and B', and the outcomes are restricted to be in the set $\{-1,1\}$. It is assumed that probabilities $p_{AB}(1,1) = p_{AB}(-1,-1) = p_{AB'}(1,1) = p_{AB}(-1,-1) = p_{A'B}(1,1) = p_{A'B}(-1,-1) = p_{A'B'}(1,-1) = p_{A'B'}(-1,1) = 1/2$, and all other probabilities are zero. Then, the CHSH quantity equals to 4. However, if we calculate the local probabilities, we will find all of them equal to $1/2$, and then we can find conditional probabilities, say $p_{B|A}(1|1) = 1$ from $p_{AB}(1,1) = p_{B|A}(1|1)p_A(1)$. We find B and B' should be the same since observations of A is perfectly correlated with both of them, yet at the same time B and B' should be opposite since observations of A' is perfectly correlated with B while anti-correlated with B' [31]. Thus, the global assignment of probabilities does not allow one consistent local assignment of probabilities, and the probability distributions of random variables of the two subsystems cannot simultaneously exist, in other words, the model is counterfactual. Our analysis further indicates that the principle of causality itself is not enough to set a physical constraint on the bound of CHSH inequality, what is missing is the notion of locality, since at least the local parts of the nonlocal box should exist, and at the same time, special relativity also is consistent with the

nature of locality.

The formalism for nonlocal box has been developed, which takes a more abstract mathematical form. Two subsystems A and B takes inputs x and y into outputs a and b respectively, $x, y \in \{0,1\}$, $a, b \in \{0,1\}$, and $a \oplus b = x \cdot y$, $\oplus$ means mod2. However, this super-quantum correlation in nonlocal box is not comparable with the quantum correlation, since there is no clear correspondence between them. There will be different bounds which might be higher if the function for the nonlocal box is changed. The physical validity of the nonlocal box has been criticized from different aspects [7, 32–34], which are consistent with our observation that the super-quantum correlation is not simulatable with the standard CHSH setting or the classical simulation model, that is, the nonlocal box is not physical. As the result, we are lead to the conclusion that the notion of nonlocality is improper.

IV. DISCUSSIONS

In this section, we discuss some implications of our study and conclude briefly.

Local, global, and relational. The existence of space-time structure for any object in nature indicates that there exists local independent reality of objects. Locality is a basic principle in nature, however, nonlocality violates the local existence and independence of objects, thus it is not a correct concept. The uncertainty principle respects locality, since without any doubt local quantum systems exist. Mathematically, the Hilbert space can exist consistently with a spacetime structure, which forms the basis for quantum field theory also quantum gravity. From our study, a violation of Bell's or CHSH inequality does not manifest nonlocality, instead, it only specifies the existence of correlations between parties of a multipartite system following from the uncertainty principle. In physics, the "nonlocal" properties are usually specified by *global* and *relational* quantities, such as the total number of particles and correlation function, respectively, while local quantities include the mass of the particle, etc. The existence of global and relational quantities relies on the existence of local quantities, thus respects locality and denies nonlocality.

Complementarity. What does the violation of Bell-type inequalities mean? From our study, the answer is not about nonlocality, and, actually it does not mean much. The notion of nonlocality was expected to capture the essence of quantum, yet it does not. Then, how can we study the property of quantum system? At present, there is no unique way. Regarding quantum state and observable, we can classify three types:

- Entanglement: property of quantum state; "observable-independent".
- Entropic uncertainty: property of observable (including measurement); "state-independent".
- Complementarity: property of the combination of quantum state and observable (including measurement).

The three frameworks above each have their own merits, and cannot replace each other. Note that the original notion of complementarity by Bohr is more philosophical rather than physical, yet as it captures the overall properties of 'quantum', it is a proper terminology to describe the property of the combination of quantum state and observable. Roughly, we can also say that entanglement and entropic uncertainty demonstrate the property of complementarity.

Hidden variable theories. It is hard to describe the hidden variable theories (HVTs) precisely [1–4, 35, 36]. The main argument of HVTs is that the wave function does not provide the complete description of a quantum system, there exists the hidden variable, say λ, which determines the randomness due to wave function. However, up to now, all the experiments respect quantum mechanics. There are mainly three kinds of theories relating to HVTs. The first one is Bell's theorem [2], which states there is no local hidden variable, demonstrated by Bell's inequality (or Bell's theorem without inequality, e.g., GHZ theorem [37], Hardy theorem [38]). The second one is Bell-Kochen-Specker (Bell-KS) theorem [39], which roles out noncontextual hidden variable for system of at least 3 dimension. The third one is Bohmian mechanics [40], where each quantum particle has a precise space-time position (coordinate) and trajectory. Strictly speaking, Bohmian mechanics is not a kind of HVT, since the position does not determine the wave function and related randomness, on the contrary, the wave function, together with the Hamiltonian and the boundary condition, determine the position. Position is rather one external classical parameter than hidden variable. From our study, we view the concerns of HVTs are not about whether there is one hidden variable or a variable set, or whether the variable is local or nonlocal, instead, it asks for a *hidden dynamics* which could explain, e.g., *what is wave function?*, *why there exists coherence?*, *is there macroscopic quantum coherence?*, *what is spin?*, to which quantum theory cannot provide the answer. In this sense we can say that quantum mechanics is not complete since not all elements of physical reality have a counterpart in it. However, every theory is not complete due to the fact that every theory is just a special kind of description of the properties of nature, for instance, classical mechanics is not complete since it does not provide the wave function. The probabilistic nature of quantum behavior manifests that quantum system has the propensity to be in different states, the wave function is a systematic (or complementary) description of motion of one object.

To conclude, we have demonstrated that the notion of nonlocality is improper, and the principle of locality is respected by the uncertainty principle. To understand quantum, different notions can be employed which include entanglement, entropic uncertainty, and complementarity (including uncertainty). Yet to explain quan-

tum, e.g. the existence of coherence, some unknown post-quantum theory which probably is not a hidden variable theory is to be discovered.

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